Roll No. ....

E–311

M. A./M. Sc. (First Semester)

EXAMINATION, Dec.-Jan., 2020-21

MATHEMATICS

Paper Third

(Topology)

Time : Three Hours] [*Maximum Marks : 80*

Note : Attempt all Sections as directed.

Section—A 1 each

(Objective/Multiple Choice Questions)

Note : Attempt all questions.

Choose the correct answer :

1. If A and B are disjoint sets, then :

   (a) $|A \cup B| = |A + B|$

   (b) $|A \cup B| = |AB|$

   (c) $|A \cup B| = |A| + |B|$

   (d) $|A \cup B| = |A|\,|B|$

2. If f is a mapping form a locally compact space X onto a housdorff space Y, then Y is also locally compact if f is :
   (a) Continuous
   (b) Open
   (c) Continuous and open
   (d) Bijective and continuous

3. Which is not true ?
   (a) $\overline{\overline{A}} = \overline{A}$
   (b) $\overline{A \cup B} = \overline{A} \cup \overline{B}$
   (c) $\overline{A \cap B} = \overline{A} \cap \overline{B}$
   (d) $A \subset B \Rightarrow \overline{A} \subset \overline{B}$

4. The topological space (X, Y), where $X = \{a, b\}, Y = \{\phi, X\{a\}\}$ is :
   (a) T_0
   (b) T_1
   (c) Neither T_0 nor T_1
   (d) Both T_0 and T_1

5. "Every non-void partially ordered set contains a maximal chain" is known as :
   (a) Zorn's lemma
   (b) Tukey's lemma
   (c) Axiom of choice
   (d) Hausdorff Maximal Principle.

6. Set R of real numbers with usual metric is :
   (a) Connected
   (b) Disconnected
   (c) Totally disconnected
   (d) None of these

7. Which is not true ?
   (a) Every compact space is countably compact.
   (b) Every compact space is locally compact.
   (c) Every locally compact space is compact
   (d) Every sequentially compact space is countably compact

8. A compact subset of a Hausdorff space is :
   (a) Compact
   (b) Closed
   (c) Open
   (d) None of these

9. Which is not true ?
   (a) Every regular space is normal
   (b) Every regular, second countable space is normal
   (c) Every compact, Hausdorff space is T_3
   (d) Every regular Lindeloff space is normal

10. Which is not true for a discrete space ?
   (a) It is locally connected
   (b) It is totally disconnected
   (c) It is disconnected
   (d) Each singleton set is a component

11. $\overline{A}$ is :

(a) Intersection of all closed subsets of A.

(b) Union of all closed subsets of A.

(c) Intersection of all closed supersets of A.

(d) Union of all closed supersets of A.

12. Each compact subset of a locally connected space is :

(a) Closed

(b) Open

(c) Neither closed nor open

(d) None of these

13. Every totally bounded metric space is :

(a) Reducible

(b) Locally compact

(c) Separable

(d) Compact

14. 'Generalized Heine–Borel Theorem states that, "A subset of R^n is" :

(a) Closed iff it is compact and bounded

(b) Compact iff it is closed and bounded

(c) Closed iff it is compact

(d) Compact iff it is closed

15. Which is countable set ?

(a) Set of all real numbers

(b) Set of all rational numbers

(c) Set of all irrational numbers

(d) The unit interval [0, 1]

16. A mapping f from a space X into space Y is continuous iff for every $\mathrm{A} \subset \mathrm{X}$:

(a) $\overline{f(\mathrm{A})} \subset f(\overline{\mathrm{A}})$

(b) $f(\mathrm{A}) \subset f(\overline{\mathrm{A}})$

(c) $\overline{f(\mathrm{A})} \subset f(\mathrm{A})$

(d) $f(\overline{\mathrm{A}}) \subset \overline{f(A)}$

17. Urysohn's lemma is related to :

(a) Normal space

(b) Separable space

(c) Compact space

(d) Hausdorff space

18. A metric space is compact if it is :

(a) Complete

(b) Totally bounded

(c) Complete and totally bounded

(d) Complete and bounded

19. Which **T** is a topology on X = {1, 2, 3, 4}

(a) $\mathbf{T} = \phi, \mathrm{X}, \{1\}, \{2\}$

(b) $\mathbf{T} = \phi, \mathrm{X}, \{1\}, \{2\}, \{3\}, \{1, 2\}$

(c) $\mathbf{T} = \phi, \{1\}, \{2\}, \{3\}, \{1, 2\}$

(d) $\mathbf{T} = \phi, \mathrm{X}\ \{1\}, \{2\}, \{1, 2\}$

20. Cantor's Theorem states that :

    (a) Every set is equivalent to its power set

    (b) No set can be equivalent to its power set

    (c) Every finite set contains a denumerable subset

    (d) Every infinite set contains a denumerable subset

Section—B $1\frac{1}{2}$ each

(Very Short Answer Type Questions)

Note : Attempt all questions.

1. Define countable set.
2. Define Axiom of Choice.
3. Define sequentially compactness.
4. Define separable space.
5. Define limit point and derived set.
6. Define Frechet space.
7. State Bolzano-Weierstrass property.
8. Define totally bounded set.
9. Define locally connected space.
10. Define Kuratowski Closure axioms.

Section—C $2\frac{1}{2}$ each

(Short Answer Type Questions)

Note : Attempt all questions.

1. Prove that every infinite subset of a countable set is countable.
2. Prove that co-finite topological space is compact.
3. Prove that every completely regular space is regular.
4. If f is a homeomorphism from topological space X to Y, then prove that f is a continous bijection and closed.
5. Prove that every metric space is a Hausdorff space.

6. Prove that sountable compactness is weakly hereditary property.
7. If λ is any infinite cardinal, then prove that :

$$\lambda + \lambda = \lambda$$

8. Prove that every continuous image of a connected space is also connected.
9. Let (X, **T**) be a topological space, where X = {*a, b, c, d, e*} and $\mathbf{T} = \phi, X, \{a\}, \{a, b\}, \{a, c, d\}, \{a, b, c, d\}\{a, b, e\}$

   Then find the neighborhoods of the point 'e' and 'c'.
10. Prove that if X is a connected space then X cannot be written as the disjoint union of two non-empty closed sets.

Section—D 4 each

(Long Answer Type Questions)

Note : Attempt all questions.

1. State and prove Schroder-Bernstein theorem.

   Or

   Prove the equivalence of Zermelo's postulate and the Axiom of choice.
2. Define topological space. Let X be a non-empty set and let **T** consists of empty set and all co-finite subsets of X. Then prove that (X, **T**) is a topological space.

   Or

   Define interior point of a set. Prove that interior of a subset A of X is the union of all open sets contained in A.
3. Prove that every regular, Lindeloff space is normal.

   Or

   State and prove Urysohn' lemma.

4. Prove that every sequentially compact space is countably compact whereas a first countable and countably compact is a sequentially compact topological space.

Or

Define one-point compactification. State and prove Stone-Cech compactification theorem.

5. State and prove Lebesgue covering lemma.

Or

Prove that closure of a connected subset is also connected.